A MODELING APPROACH TO TRANSITIONAL CHANNEL FLOW

R. A. Sahan, H. Gunes, A. Liakopoulos
Department of Mechanical Engineering and Mechanics
Lehigh University
Bethlehem, PA 18015-3085, USA

ABSTRACT

Low-dimensional dynamical models of transitional flow in a periodically grooved channel are numerically obtained. The governing partial differential equations (continuity and Navier-Stokes equations) with appropriate boundary conditions are solved by a spectral element method for Reynolds number $Re = 430$. The method of empirical eigenfunctions (Proper Orthogonal Decomposition) is then used to extract the most energetic velocity eigenmodes, enabling us to represent the velocity field in an optimal way. The eigenfunctions enable us to identify the spatio-temporal (coherent) structures of the flow as travelling waves, and to explain the related flow dynamics. Using the computed eigenfunctions as basis functions in a truncated series representation of the velocity field, low-dimensional models are obtained by Galerkin projection. The reduced systems, consisting of few non-linear ordinary differential equations, are solved using a fourth-order Runge-Kutta method. It is found that the temporal evolution of the most energetic modes calculated using the reduced models are in good agreement with the full model results. For the modes of lesser energy, low-dimensional models predict typically slightly larger amplitude oscillations than the full model. For the slightly supercritical flow at hand, reduced models require at least four modes (capturing about 99% of the total flow energy). This is the smallest set of modes capable of predicting stable, self-sustained oscillations with correct amplitude and frequency. POD-based low-dimensional dynamical models considerably reduce the computational time and power required to simulate transitional open flow systems.

1 INTRODUCTION

A large number of engineering applications deal with flow in periodically repeated configurations such as flow over grooves and flow in grooved channels [1-6]. The grooved channel geometry, shown in Fig. 1, is encountered in many low-speed applications such as cooling of electronic devices and circuit boards [5-6], and in flow and heat transfer in compact heat exchangers [4]. In small size systems with moderate Reynolds number, Re, transport enhancement is important and is achieved by mixing due to hydrodynamic instabilities [6]. The grooved channel geometry represents one such configuration characterized by wall bounded flow with separation. The fluid flow in the channel can be divided into two parts: the by-pass and the groove regions (see Fig. 2). The flow patterns in the by-pass and groove regions differ greatly. Approximately parallel flow structure is present in the by-pass region which is separated from the recirculating zones within the grooved regions by shear layers [5-6]. The shear layer partitioning the flow regions does not permit convective exchange of fluid. The combination of the above factors results in differences in convective exchange of fluid between the regions. The overall flow reaches a time-dependent state when the Reynolds number exceeds a critical value Re_c. When $Re_c < Re < Re_2$, the flow exhibits self-sustained, time-periodic oscillations [5-6].

Direct numerical simulations (DNSs) of transitional flow in grooved channel geometries have been performed in order to investigate both flow and heat transfer characteristics [2,3,5,6]. However, stability, bifurcation and flow control studies by DNS require tremendous computational resources. The necessity for low-dimensional models (LDMs), which carry considerable physical information on the dynamical behavior of the flow, arises in order to reduce the size of the problem and to make stability, bifurcation and control studies of transitional flows in complex configurations feasible.

The modeling of partial differential equations (PDEs) by minimal systems of ordinary differential equations (ODEs) has become a major issue in the study of complicated flow phenomena such as transition and turbulence. LDMs can replace the system of governing PDEs by a small set of non-linear ODEs. Since DNS of transitional and turbulent flows in complex geometries requires tremendous amount of computational time, an accurate low-dimensional approximation allows us to perform parametric studies of such flows with considerably less computational effort [7-15]. Spectral methods (such as Fourier-Galerkin, Chebyshev-Galerkin, etc. proposed by Gotlieb and Orszag [16]) or weighted residuals techniques based on splines (e.g., Liakopoulos and Hsu [17], Liakopoulos [18]) may be used to transform a PDE into a system of ODEs. However, these methods are very general and do not lead to minimal systems (see for example the discussion in Liakopoulos et al. [13]).

Proper Orthogonal Decomposition (POD), also known as Karhunen-Loeve Decomposition, was introduced by Lumley [19] in order to identify coherent structures in isothermal turbulent flows. The method can also be used to extract the empirical eigenfunctions (and the related spatio-temporal structures) of transitional flows. Sirovich [20] proposed the snapshot version of POD in order to analyze large data sets with less computational effort. Berkooz et al. [21] and Holmes et al. [22] provide a thorough review of the important aspects of POD as applied to the analysis of turbulent flows. A large number of POD applications and POD-based LDMs have been reported in the last decade. Aubry et al. [23] and Zhou and Sirovich [24] applied POD to obtain LDMs of the near wall region of a turbulent boundary layer. Sirovich et al. [25], Deane and Sirovich [26] and Tarman [27] applied POD to turbulent Rayleigh-Benard convection. Deane et al. [7] analyzed complex geometry flows in the transitional regime while Deane and Mavriplis [28] used the method in a low-dimensional description of the dynamics of separated unsteady flow past thick airfoils. Rajaee et al. [29] employed POD in the study of free-shear-flow coherent structures and their dynamical behavior in order to obtain a low-dimensional description of the flow. Rempfer and Fasel [30-31] and Rempfer [32] used the method in studying the evolution of three dimensional coherent structures of a transitional, spatially evolving flat-plate boundary layer. Newman and Karniadakis [33] developed LDMs of flow-induced vibrations via POD. Sahan et al. [8-9] used POD to obtain LDMs of non-isothermal flow in a periodically grooved channel while Gunes et al. [10-12] developed reduced models of buoyancy-induced flow in a vertical channel. Stability and bifurcation issues related to POD-based low-dimensional models are discussed in Liakopoulos et al. [13].

In the present study, snapshot POD is used to extract the coherent structures and to derive low-dimensional dynamical models of a two-dimensional transitional flow in a periodically grooved channel. Using the empirical eigenfunctions, reconstruction of the original flow is obtained in an optimal way. A low-dimensional set of nonlinear ordinary differential equations that describes the dynamics of the flow field is derived by Galerkin projection. The paper consists of five sections. In Section 2, the full model governing equations and the solution method are briefly discussed. In Section 3, the development of LDMs via POD is described and relevant properties of the empirical eigenfunctions are presented. POD results and low-dimensional dynamical model predictions are discussed in Section 4. Concluding remarks are summarized in Section 5.

2 FULL MODEL EQUATIONS

2.1 Formulation of the problem

The grooved channel geometry under study is shown in Fig. 1. The channel is periodic in the streamwise direction and infinite in depth. The flow is assumed to be periodically fully-developed and the fluid is considered to be incompressible with constant transport properties. The governing PDEs for two-dimensional time-dependent isothermal flow can be written in dimensionless form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right] + F_x, \tag{2}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right] + F_y. \tag{3}$$

The independent and dependent variables have been non-dimensionalized using the following definitions

$$(X,Y) = \frac{1}{h_1}(x,y), \quad (U,V) = \frac{1}{U_{ref}}(u,v), \quad P = \frac{p}{U_{ref}^2 p}$$

$t=\frac{U_{ref}}{\mathrm{h}_1}\,t^*,\;\; F_{\mathrm{x}}=\frac{\mathrm{h}_1}{U_{ref}^2}f_x, \quad F_{\mathrm{y}}=\frac{\mathrm{h}_1}{U_{ref}^2}f_y..$

In general, lower case letters denote dimensional variables. However, note that the dimensional time is denoted by t^*. In the above, (x, y) and (u, v) denote the Cartesian coordinates and velocity components, respectively, p is the static pressure, $2h_1$ is the width of the by-pass portion of the channel (see Fig. 2), ν is the kinematic viscosity, ρ is the fluid density and $U_{ref} = (3/2)U_{av}$ denotes the reference velocity where U_{av} is the average velocity at a channel cross-section. F_x and F_y are the forcing terms in x and y directions, respectively, and Re =(U_{ref} h_1)/ν denotes the Reynolds number. The boundary conditions of the problem are:
(i) No slip conditions at the solid-fluid interfaces,

$$V(X, Y, t) = 0, \tag{4}$$

(ii) Periodic boundary conditions in the streamwise direction,

$$V(0, Y,\, t) = V(l_1, Y,\, t), \quad 0 < Y < 2h_1. \tag{5}$$

In the above equations, l_1 denotes the dimensionless length of the computational domain in the streamwise direction, and l_2 is the spacing between the modules (see Fig. 2(a)). Note that since periodic boundary conditions are imposed in the x-direction, the model is valid far from the channel entrance.

2.2 Method of solution

A spectral element method, Patera [34], is used to solve the governing equations (1)-(3) with the boundary conditions (4)-(5). Implementation of the method is based on Nekton, a computer code developed for the simulation of steady and unsteady incompressible fluid flow, heat and mass transfer. We refer to Ref. [35] for a detailed discussion of the numerical method. In our simulations, 44 spectral elements are used (See Fig. 2(b)). Numerical solutions were obtained for order of interpolants, N= 4, 6, 8, 10 and 12. The error due to spatial resolution is found to be small for N$\geq$ 8. Computations were performed on a Stardent P3000 computer and an IBM RS/6000 Model R24.

3 DEVELOPMENT OF LOW-ORDER MODELS

As mentioned earlier, POD is used to obtained reduced dynamical models of the flow system. POD selects an orthogonal set of spatial modes that is optimal in terms of retained kinetic energy. In applying the snapshot POD [20], the time-dependent data obtained by DNS are decomposed into time-averaged ($\overline{U}$, $\overline{V}$) and time-varying (U', V') parts. The time-averaged flow field is obtained as the simple arithmetic mean of M snapshots. The empirical eigenfunctions are computed as linear combination of the time-varying parts of the field variable [20], i.e., in component form,

$$\phi_{uk}(X,Y) = \sum_{i=1}^{M} A_{ki} U_i'(X,Y,t_i), \quad (6)$$

$$\phi_{vk}(X,Y) = \sum_{i=1}^{M} A_{ki} V_i'(X,Y,t_i), \quad (7)$$

where A_k denotes the eigenvectors of the matrix eigenvalue problem

$$CA = \lambda \ A, \quad (8)$$

and the elements of matrix C are

$$C_{mn} = \frac{1}{M} \int_\Omega \left[U_m'(X,Y,t_m) \ U_n'(X,Y,t_n) + V_m'(X,Y,t_m) \ V_n'(X,Y,t_n) \right] d\Omega. \quad (9)$$

The resulting eigensystem has the following properties [20]: (i) the eigenfunctions are orthogonal to each other, (ii) the eigenfunctions satisfy the boundary conditions of the fluctuation field, (iii) the empirical eigenfunctions are divergence-free since the flow is incompressible and (iv) the eigenvalues are real and non-negative. The eigenvalues are ordered as $\lambda_1 \geq \lambda_2 \geq \lambda_3 ... \geq \lambda_n$, $n = 1, 2, ..., M$. After normalizing the eigenfunctions, the time-varying part of the velocity field is expanded in terms of the eigenfunctions as,

$$U'(X,Y,t) = \sum_{k=1}^{M_1} a_k \ (t) \ \phi_{uk}(X,Y), \quad (10)$$

$$V'(X,Y,t) = \sum_{k=1}^{M_1} a_k \ (t) \ \phi_{vk}(X,Y), \quad (11)$$

where ϕ_{uk} and ϕ_{vk} denote the the x- and y- components of the kth eigenfunctions of the velocity field. In general, $M_1 \ll M$ in order to keep the dimensionality of the reduced model low. The instantaneous velocity field components can now be expressed as

$$U(X,Y,t) = \overline{U}(X,Y) + \sum_{k=1}^{M_1} a_k(t) \ \phi_{uk}(X,Y), \quad (12)$$

$$V(X,Y,t) = \overline{V}(X,Y) + \sum_{k=1}^{M_1} a_k \ (t) \ \phi_{vk}(X,Y). \quad (13)$$

Substituting equations (12)-(13) into the governing equations (1)-(3), applying Galerkin projection and making use of the orthonormality property of the empirical eigenfunctions, a system of non-linear ODEs for the temporal expansion coefficients $a_k(t)$ of the form

$$\frac{da_k}{dt} = A_k + \tfrac{1}{Re} B_k + \sum_{i=1}^{M_1} C_{ki} \ a_i + \tfrac{1}{Re} \sum_{i=1}^{M_1} D_{ki} \ a_i + \sum_{i=1}^{M_1} \sum_{j=1}^{M_1} E_{kij} \ a_i a_j \quad (14)$$

is obtained. The coefficients (A_k, B_k, C_{ki}, D_{ki} and E_{kij}) appearing in the above equations are related to the various inner products among the eigenmodes and/or mean flow field. There is no contribution from the pressure term in the above equation [7]. Equation (14) is integrated numerically using a fourth order Runge-Kutta solver. Convergence is checked by reducing the integration time step.

4 RESULTS AND DISCUSSION

Results are presented for a six-mode model obtained by retaining the six most energetic velocity eigenfunctions. The decomposition is performed at $Re = 430$. At this Reynolds number, the long term attractor of the flow in the periodically grooved channel under consideration corresponds to a limit cycle. We take $M = 20$ snapshots in a period. Similar results are obtained for larger sets of snapshots, e.g., $M = 30$, 40 and 50. Aspect ratios are $\frac{h_2}{h_1} = 0.75$, $\frac{l_1}{h_1} = 5.0$ and $\frac{l_2}{h_1} = 3.0$. Instantaneous velocity vector and streamlines are shown in Fig. 3 for $Re = 430$. Calculation of the flow in two adjacent modules (instead of one module as shown in Fig. 3) does not change the solution.

4.1 Eigenvalues

The eight largest normalized eigenvalues, corresponding to the most energetic modes, are given in Table 1 in descending order. The far right column shows the cumulative sum of the eigenvalues. The modes occur in pairs. The eigenvalues within each pair are comparable in magnitude (see Table 1). This pattern is identical to the ones that Deane et al. [7] and Sahan et al. [8-9] found in their work.

The first two modes are dominant, providing the highest contribution to the total flow energy. Table 1 reveals that the first two modes capture about 98.1% of the entire energy of the velocity field. The cumulative contribution from the first four modes reaches the energy level of 99.7%, suggesting the dominance of just a few primary modes. The first six modes capture practically the total energy of the flow. The same observations can be made by examining Fig. 4.

4.2 Eigenfunctions and spatio-temporal (coherent) structures

The velocity empirical eigenfunctions, $\phi_i = \phi_{ui}\vec{i} + \phi_{vi}\vec{j},\ i = 1, 2, 3, 4$, are shown in Figs. 5-7. The minimum and maximum values of the stream functions corresponding to the velocity empirical eigenfunctions are listed in Table 2. As observed in the figures, the two most energetic modes contain the larger scale features of the flow while the modes with smaller eigenvalues (lower fluctuation kinetic energy) capture smaller scale features. As noted by Deane et al. [7] and Sahan et al. [8-9] and as can be observed in Figs. 5-7, eigenmodes come in pairs. Within a pair, the spatial structures are phase shifted in the streamwise direction by approximately a quarter-wavelength. Fig. 8 reveals that the corresponding temporal expansion coefficients are also phase shifted in time by a quarter of a period, so that, if we multiply the eigenfunctions of a pair by their corresponding temporal expansion coefficients, we obtain a structure that is moving in the streamwise direction. Although the structures represented by the eigenfunctions are fixed in space, when one mode within a pair is in the maximum energy state the other is in the minimum energy state and this relation reverses after one quarter of a period in time (see Fig. 8). This observation is true for all the pairs of eigenfunctions. The eigenfunctions are related to the coherent (spatio-temporal) structures of the flow and each eigenfunction can be considered as a part of a coherent structure at an instant of time. Coherent structures of the flow change in shape and energy as they move in the streamwise direction. The dynamical coherent structures, $\xi_j(X, Y, t)$, for transitional grooved channel flow can be represented by pairs of eigenfunctions with almost identical eigenvalues. For example, the following definition represents the first order coherent structure of velocity

$$\xi_1(X,Y,t) = \phi_{u1}(X,Y)\,a_1(t)\vec{i} + \phi_{v1}(X,Y)\,a_1(t)\vec{j}$$

$$+\phi_{u2}(X,Y)\,a_2(t)\vec{i} + \phi_{v2}(X,Y)\,a_2(t)\vec{j}. \tag{15}$$

As observed, a pair of eigenfunctions/temporal coefficients not only contains complete information on the typical shape of the corresponding coherent structure, but also shows the evolution that the coherent structure undergoes while moving in the streamwise direction (see Figs. 9-10).

4.3 Reconstruction of the flow field variables and optimization of the mode retaining process

Sahan et al. [8-9] discussed the issue of optimal number of eigenmodes retained in the LDMs. In order to achieve a useful LDM, one should: (i) keep as few modes as possible in order to reduce the dimensionality of the developed model, ii) retain enough modes so that the flow field variable, i.e., the velocity field, is reconstructed accurately and a proper dynamical representation of the flow system is obtained and iii) capture most of the flow fluctuation energy without losing the potentially critical information hidden in the higher modes. However it should be mentioned that although the reconstruction error is considerably reduced as the number of retained modes increases, inclusion of relatively noisy higher eigenmodes may cause significant performance loss for the low-dimensional models.

It is obvious that we wish to keep as few modes as possible in a low-dimensional system, permitting us to apply the techniques of dynamical system analysis. On the other hand, we would like to retain at least a qualitatively correct dynamical representation of the transitional flow inside the grooved channel. Although retaining two or three modes in the truncated series expansion approximates the flow field successfully, capturing about 99% of the total energy, the first three modes can not form an adequate basis for the development of a valid LDM. The resulting LDM either does not produce stable oscillations in time or fails to predict the correct amplitude and frequency of oscillations. Keeping four, five or six modes in the truncated series expansion leads to LDMs that predict correctly both the amplitude and frequency of the oscillations. In the present study, it is found that although retaining more than six modes captures the entire flow energy (about 99.9%), the resulting LDM may not predict the flow system's long-time dynamical behavior accurately. If the noise introduced by higher modes is considerable, the amplitudes and frequencies of the temporal expansion coefficients are not estimated correctly. This explains why the results of the six-mode dynamical model are presented in this paper.

4.4 Low-dimensional model predictions

Figure 8 shows the variation of the six expansion coefficients obtained by direct projection of the input velocity data (full model) on the computed eigenfunctions. Temporal expansion coefficients predicted by the six-mode model are shown in Fig. 11 together with the direct projection results. The results for the first four temporal modes are in good agreement (see Fig. 11). However, in general, predictions for higher modes exhibit oscillations of slightly larger amplitude than those calculated by direct projection. It is also observed that temporal modes calculated using LDM have slightly larger period than those calculated by direct projection. Similar results are obtained by using either a four-mode or a five-mode LDM. As mentioned earlier, if the order of the system of ODEs is further reduced, the resulting models do not estimate the system's dynamical behavior correctly. Comparisons between full simulation data and the long-time six-mode model predictions in phase-space are shown in Fig. 12.

5 CONCLUDING REMARKS

Two-dimensional transitional isothermal flow in a periodically grooved channel has been numerically investigated and low-dimensional dynamical models have been developed. A time-dependent solution in the transitional regime has been analyzed by the method of empirical eigenfunctions (POD) and empirical eigenfunctions and spatio-temporal structures of the flow have been extracted. The eigenfunctions associated with the largest eigenvalues are the modes that contain the largest fraction of the flow energy and explain the dynamical behavior of the flow. Sets of ODEs were derived for the time-dependent modal amplitudes.

The first six velocity modes contain almost all the flow fluctuation energy. These modes occur in pairs and are phase-shifted both in space and time, corresponding to travelling waves. These six modes reconstruct the flow successfully. The predictions of the six-mode model was compared with the full model results. Amplitudes of the first two modal pairs are in good agreement. However, for higher modes, the six-mode LDM predicts mode amplitudes that are slightly larger than those obtained by direct projection. Keeping less than four modes in the truncated series expansion either does not produce stable self-sustained oscillations in time or fails to predict the correct amplitude of the oscillations.

Low-dimensional models of transitional isothermal flow reduce dramatically the size of the computational problem. In addition, POD-based reduced models may enable us to make stability, bifurcation and control analysis of the flow system feasible with less computational effort and storage requirements. In order to utilize the developed models in real-time flow control applications, we have recently explored intelligent control strategies based on the derived LDMs and the modeling and fast processing capabilities of artificial neural networks [14,36].

REFERENCES

[1] N.K. Ghaddar, K.Z. Korczak, B.B. Mikic and A.T. Patera, Numerical investigation of incompressible flow in grooved channels. part 1. stability and self-sustained oscillations. J. Fluid Mech. 163, 99 (1986).

[2] C.H. Amon and A.T. Patera, Numerical calculation of stable three-dimensional tertiary states in grooved-channel flow. Phys. Fluids A 1, 2005 (1989).

[3] G.E. Karniadakis, Spectral element simulation of laminar and turbulent flows in complex geometries. Appl. Num. M. 6, 85 (1989).

[4] C.H. Amon and B.B. Mikic, Numerical prediction of convective heat transfer in self sustained oscillatory flows. J. Thermophys. Heat Transfer. 4, 239 (1990).

[5] J.S. Nigen and C.H. Amon, Forced convective cooling enhancement of surface mounted electronic package configurations through self-sustained oscillatory flows. Heat Transfer in Electronic Equipment, ASME-HTD 171, 39 (1991).

[6] J.S. Nigen and C.H. Amon, Forced convective cooling enhancement of electronic package configurations through self-sustained oscillatory flows. ASME J. Electron Packaging, 115, 356 (1993).

[7] A.E. Deane, I.G. Kevrekidis, G.E. Karniadakis and S.A. Orszag, Low dimensional models for complex geometry flows: application to grooved channels and circular cylinders. Phys. Fluids A 3, 2337 (1991).

[8] R.A. Sahan, H. Gunes and A. Liakopoulos, Low-dimensional models for coupled momentum and energy transport problems. In Cooling and Thermal Design of Electronic Systems (Edited by C. Amon), ASME-HTD 319, 1 (1995).

[9] R.A. Sahan, A. Liakopoulos and H. Gunes, Reduced dynamical models of nonisothermal transitional grooved-channel flow. Phys. Fluids A 9, 551 (1997).

[10] H. Gunes, R.A. Sahan and A. Liakopoulos, Low-dimensional representation of buoyancy driven flow in a vertical channel with discrete heaters. In Enhancing Natural Convection Cooling of Electronic Systems and Components (Edited by A. Ortega and S.P. Mulay), ASME-HTD 303, 125 (1995).

[11] H. Gunes, A. Liakopoulos and R.A. Sahan, Low-dimensional description of oscillatory thermal convection: the small Prandtl number limit. Theoret. Comput. Fluid Dyn. in-press (1997).

[12] H. Gunes, R.A. Sahan and A. Liakopoulos, Spatio-temporal structures of buoyancy-induced flow in a vertical channel. Numer. Heat Transfer: Part A. Appl. in-press (1997).
[13] A. Liakopoulos, P.A. Blythe, and H. Gunes, A reduced dynamical model of convective flows in tall laterally heated cavities. Proc. Royal Society London A. 453, 663 (1997).
[14] R.A. Sahan, Low-order dynamical modeling and intelligent control of thermo-fluid systems via proper orthogonal decomposition. Ph.D. Dissertation, Lehigh University, Mechanical Engineering and Engineering Mechanics Department, Bethlehem, PA, USA (1997).
[15] H. Gunes, Low-dimensional models of transitional convective flows. Ph.D. Dissertation, Lehigh University, Mechanical Engineering and Engineering Mechanics Department, Bethlehem, PA, USA (1997).
[16] D. Gottlieb and S. Orszag, Numerical Analysis of Spectral Methods, Society for Industrial and Applied Mathematics, Philadelphia (1977).
[17] A. Liakopoulos and C.C. Hsu, On a class of compressible laminar boundary-layer flows and the solution behavior near separation. J. Fluid Mech. 149, 339 (1984).
[18] A. Liakopoulos, Computation of high speed turbulent boundary-layer flows using the k-ϵ turbulence model. Int. J. Numer. Meth. Fluids 5, 81 (1985).
[19] J.L. Lumley, The structure of inhomogeneous turbulent flow. In Atmospheric Turbulence and Radio Wave Propagation (Edited by A.M. Yaglom and V.I. Tatarski), p. 160. Nauko, Moscow (1967).
[20] L. Sirovich, Turbulence and dynamics of coherent structures: I-III. Q. Appl. Math. 45, 561 (1987).
[21] G. Berkooz, P. Holmes and J.L. Lumley, The proper orthogonal decomposition in the analysis of turbulent flows. Ann. Rev. Fluid Mech. 25, 539 (1993).
[22] P. Holmes, J.L. Lumley and G. Berkooz, Turbulence, Coherent Structures, Dynamical Systems and Symmetry, Cambridge University Press, Great Britain (1996).
[23] N. Aubry, P. Holmes, J.L. Lumley and E. Stone, The dynamics of coherent structures in the wall region of a turbulent boundary layer. J. Fluid Mech. 192, 115 (1988).
[24] X. Zhou and L. Sirovich, Coherence and chaos in a model of turbulent layer. Phys. Fluids A 4, 2855 (1992).
[25] L. Sirovich, M. Maxey and H. Tarman, An eigenfunction analysis of turbulent thermal convection. In Turbulent Shear Flows 6 (Edited by J.C. Andre, J. Cousteix and F. Durst et al.), p. 68. Springer, New York (1989).
[26] A.E. Deane and L. Sirovich, A computational study of Rayleigh-Benard convection. part I. Rayleigh-number scaling. J. Fluid Mech. 222, 231 (1991).
[27] I.H. Tarman, A Karhunen-Loeve analysis of turbulent thermal convection. Int. J. Num. Meth. Fluids 22, 67 (1996).
[28] A.E. Deane and C. Mavriplis, Low-dimensional description of the dynamics in separated flow past thick airfoils. AIAA J. 32, 1222 (1994).

[29] M. Rajaee, S.K.F. Karlsson and L. Sirovich, Low-dimensional description of free-shear-flow coherent structures and their dynamical behavior. J. Fluid Mech. 258, 1 (1994).
[30] D. Rempfer and H.F. Fasel, Evolution of three-dimensional coherent structures in a flat-plate boundary layer. J. Fluid Mech. 260, 351 (1994).
[31] D. Rempfer and H.F. Fasel, Dynamics of three-dimensional coherent structures in a flat-plate boundary layer. J. Fluid Mech. 275, 257 (1994).
[32] D. Rempfer, Investigations of boundary layer transition via Galerkin projections on empirical eigenfunctions. Phys. Fluids A 8, 175 (1996).
[33] D. Newman and G. Karniadakis, Low-dimensional modeling of flow-induced vibrations via proper orthogonal decomposition. Report No: 96-6, Center for Fluid Mechanics, Turbulence and Computation, Brown University, Providence, RI, USA (1996).
[34] A. T. Patera, A spectral element method for fluid dynamics: laminar flow in a channel expansion. J. Comput. Phys. 54, 468 (1984).
[35] Nekton Users Manual, Release 2.9, Nektonics Inc., CT, USA (1994).
[36] R.A. Sahan, D.C. Albin, N.K. Sahan and A. Liakopoulos, Artificial neural network-based low-order dynamical modeling and intelligent control of transitional flow systems. in preparation (1997).

LIST OF TABLES:

Table 1. The eight largest normalized eigenvalues and their cumulative contribution to the total flow fluctuation energy.

mode	seNormalized Eigenvalue	Cumulative Energy, %
1	0.51395	51.39
2	0.46741	98.14
3	0.00829	98.96
4	0.00767	99.72
5	0.00117	99.84
6	0.00113	99.95
7	0.00016	99.97
8	0.00015	99.99

Table 2. Normalized velocity empirical eigenfunctions. Maximum and minimum stream function values.

	$\vec{\phi}_1$	$\vec{\phi}_2$	$\vec{\phi}_3$	$\vec{\phi}_4$
$\vec{\phi}max$	0.1089	0.1113	0.0738	0.1014
$\vec{\phi}min$	−0.1082	−0.1047	−0.0992	−0.0851

LIST OF FIGURES:

Fig. 1. Grooved channel geometry.

Fig. 2. (a) Computational domain and boundary conditions. $\dfrac{h_2}{h_1} = 0.75$, $\frac{l_1}{h_1} = 5.0$, $\frac{l_2}{h_1} = 3.0$. (b) Computational mesh. 44 spectral elements each with 9×9 collocation points.

Fig. 3. (a) Instantaneous velocity vector field. (b) Instantaneous streamlines. $Re = 430$.

Fig. 4. Contribution of modes to the total flow fluctuation energy.

Fig. 5. Velocity empirical eigenfunctions.

Fig. 6. x-component of the velocity empirical eigenfunctions.

Fig. 7. y-component of the velocity empirical eigenfunctions.

Fig. 8. Temporal expansion coefficients computed by direct projection of the snapshots on the eigenfunctions.

Fig. 9. Reconstructed x-component of the dominant spatio-temporal structure. T =period of oscillation.

Fig. 10. Reconstructed y-component of the dominant spatio-temporal structure. T =period of oscillation.

Fig. 11. Comparison of low-dimensional model predictions with full-model results. Solid line: six-mode LDM predictions. Dotted line: direct projection (full model).

Fig. 12. Phase plots of velocity temporal expansion coefficients. Solid line: six-mode LDM.

Dotted line: direct projection (full model).