Pergamon

Int. Comm. Heat Mass Transfer, Vol. 22, No. 6, pp. 837–847, 1995

0735-1933/95 $9.50 + .00

0735-1933(95)00072-0

THE ROLE OF VARIABLE VISCOSITY IN THE STABILITY OF CHANNEL FLOW

A. Pinarbasi and A. Liakopoulos
Department of Mechanical Engineering and Mechanics
Lehigh University, Bethlehem, PA, 18015

(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

The stability of plane Poiseuille flow is studied for liquids exhibiting exponential viscosity-temperature dependence. In contrast to previously published studies, viscosity and temperature fluctuations are included in the formulation. Equations describing the evolution of small, two-dimensional disturbances are derived and the stability problem is formulated as an eigenvalue problem for a set of two ordinary differential equations. A Chebyshev collocation discretization method leads to a generalized matrix eigenvalue problem which is solved by the QZ algorithm. It is found that an imposed wall temperature difference, $\Delta\bar{T}$, is always destabilizing. The instability region in the wavenumber-Reynolds number plane grows considerably as $\Delta\bar{T}$ increases. The influence of Prandtl number, temperature fluctuations and viscosity fluctuations on the flow stability/instability is small. However, their influence on the margin of stability for small wavenumbers is appreciable.

Introduction

The hydrodynamic stability of viscous flow between parallel plates has been investigated by numerous authors. However, few investigators have taken into account the effect of strongly nonisothermal conditions even though variable properties can significantly affect the critical Reynolds number and the stability characteristics of the flow. Thomas [1] and Orszag [2] considered isothermal flow conditions and solved the classical Orr-Sommerfeld stability equation numerically. Potter and Graber [3] and Wazzan *et al.* [4] studied the stability of plane Poiseuille flow and stagnation water boundary layer, respectively, including the effect of temperature dependent viscosity. In both studies, viscosity and temperature fluctuations were neglected.

Recently, Herwig and Schäfer studied the stability of two-dimensional external boundary layers [5] and plane Poiseuille flow [6] including the effect of small temperature and pressure variations on the physical properties. Their formulation adopts a Taylor series expansion of the physical properties with respect to temperature and pressure. They found that the stability characteristics of boundary layers change moderately when temperature fluctuations are included and that for a certain amount of heating, the stabilizing/destabilizing effect for water is much stronger than for air.

In this study, we address the question of variable viscosity effects on the shear instability of plane Poiseuille flow. Base flow equations and perturbation evolution equations are derived by taking into account the variation of viscosity with temperature as well as temperature and viscosity fluctuations. A linear temperature profile and an exponential viscosity law are considered. The stability problem is formulated as an eigenvalue problem for a set of ordinary differential equations. Discretization is performed using a collocation method based on Chebyshev polynomial expansions. The resulting generalized matrix eigenvalue problem is solved using the QZ algorithm.

Governing Equations

We consider two-dimensional, nonisothermal flow of an incompressible, Newtonian fluid between parallel plates. The flow is driven by a constant pressure gradient acting along the channel axis. Fluid viscosity depends on the local temperature and consequently an imposed temperature gradient gives rise to viscosity variation. The continuity, momentum balance and energy balance equations (neglecting viscous dissipation) can be written as

$$\frac{\partial \bar{u}_i}{\partial \bar{x}_i} = 0\,, \tag{1}$$

$$\bar{\rho}\left(\frac{\partial \bar{u}_j}{\partial \bar{t}} + \bar{u}_i\,\frac{\partial \bar{u}_j}{\partial \bar{x}_i}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}_j} + \frac{\partial \bar{\tau}_{ij}}{\partial \bar{x}_i}\,,\quad j = 1,2, \tag{2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}_i\,\frac{\partial \bar{T}}{\partial \bar{x}_i} = \frac{\bar{k}}{\bar{\rho}\bar{c}_p}\,\frac{\partial}{\partial \bar{x}_i}\left(\frac{\partial \bar{T}}{\partial \bar{x}_i}\right) \tag{3}$$

where the summation convention is used over index i. In the equations above, $\bar{\rho}$ denotes the fluid density, $\bar{k}$ is the thermal conductivity, $\bar{c}_p$ is the specific heat capacity at constant pressure, $\bar{p}$ is the pressure, $\bar{u}_i$ is the i-th component of the velocity vector, $\bar{\tau}_{ij}$ is the (i,j)-th component of the deviatoric stress tensor and $\bar{T}$ denotes the local temperature. For Newtonian, incompressible liquids,

$$\bar{\tau}_{ij} = \bar{\mu}\left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i}\right),$$

and the empirical viscosity-temperature relationship is of the form

$$\bar{\mu} = C\bar{\mu}_o \exp(\bar{d}/\bar{T}) . \tag{4}$$

In Eq. (4), $\bar{\mu}_o$ is the viscosity calculated at the reference temperature while C and $\bar{d}$ are constants determined from viscosity-temperature empirical curves.

Introducing dimensionless variables based on the average velocity, $\bar{U}_m$, and the channel thickness, $\bar{l}$, i.e.,

$$x_i = \frac{\bar{x}_i}{\bar{l}} , \quad t = \frac{\bar{t}\bar{U}_m}{\bar{l}} , \quad u_i = \frac{\bar{u}_i}{\bar{U}_m} , \quad p = \frac{\bar{p}}{\bar{\rho}\bar{U}_m^2} , \quad T = \frac{\bar{T}}{\bar{d}} , \quad \mu = \frac{\bar{\mu}}{\bar{\mu}_o} ,$$

the flow governing equations are transformed to,

$$\frac{\partial u_i}{\partial x_i} = 0 , \tag{5}$$

$$\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} = - \frac{\partial p}{\partial x_j} + \frac{1}{Re} \frac{\partial \tau_{ij}}{\partial x_i} , \quad j = 1, 2 , \tag{6}$$

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{1}{RePr} \frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i} \right) . \tag{7}$$

In Eq. (6),

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) ,$$

$$\mu = C \exp(1/T) , \tag{8}$$

and the Reynolds and Prandtl numbers are defined as $Re = \bar{\rho}\bar{U}_m\bar{l}/\bar{\mu}_o$ and $Pr = \bar{\mu}_o\bar{c}_p/\bar{k}$. In the remaining part of this paper we switch to conventional x, y notation. The x-axis is located at the lower channel wall.

Base Temperature and Flow Fields

The undisturbed, steady, base flow is assumed to be fully developed. Considering a temperature field that is independent of x, the energy balance equation (neglecting viscous dissipation, internal heat generation and variable thermal conductivity effects) yields a linear temperature profile. This solution of the energy equation is realizable only for isothermal channel walls. Denoting the upper hot wall temperature by $\bar{T}_h$ and the lower cold wall temperature by $\bar{T}_c$, the temperature distribution becomes

$$T_b(y) = ay + b \tag{9}$$

where $a = \Delta\bar{T}/\bar{d}$, $b = \bar{T}_c/\bar{d}$ and $\Delta\bar{T} = \bar{T}_h - \bar{T}_c$. In this study, the temperature of the cold wall is chosen as the reference temperature in viscosity calculations.

The x-component of the momentum equation takes the form

$$\left[\exp\left\{ \frac{1}{ay + b} \right\} u_b' \right]' = K , \tag{10}$$

where $K = (Re/C)(dp/dx)$, dp/dx is the dimensionless pressure gradient, u_b denotes the base velocity distribution and primes denote differentiation with respect to y. The constants C and $\bar{d}$ depend on the fluid and reference temperature chosen.

Applying the no-slip boundary conditions at the channel walls, the solution of the differential equation (10) is found to be

$$u_b(y) = \frac{K}{2a^2}\Bigg\{(ay+b)(G-b-1+ay)\exp\left(-\frac{1}{ay+b}\right) + b(b+1-G)\exp\left(-\frac{1}{b}\right) + (G-2b-1)\left[E(\frac{1}{b}) - E(\frac{1}{ay+b})\right]\Bigg\}, \tag{11}$$

where,

$$G = \frac{(a+b)(b+1-a)e^{-\frac{1}{a+b}} - b(b+1)e^{-\frac{1}{b}} + (2b+1)\left[E\left(\frac{1}{b}\right) - E\left(\frac{1}{a+b}\right)\right]}{(a+b)e^{-\frac{1}{a+b}} - be^{-\frac{1}{b}} + E\left(\frac{1}{b}\right) - E\left(\frac{1}{a+b}\right)}$$

and $E(x) = \int_x^\infty \frac{e^{-v}}{v}\,dv$ is the exponential integral (Abramowitz and Stegun [7] and Potter and Graber [3]). The pressure gradient dp/dx in the base velocity equation is found by using the condition $\int_0^1 u_b(y)dy = 1$.

Linear Stability Analysis

In this section, we formulate the stability problem by the method of small disturbances. The base velocity, pressure, temperature and viscosity fields are perturbed by imposing two-dimensional, infinitesimal disturbances, i.e.,

$$u(x,y,t) = u_b(y) + \widehat{u}(x,y,t), \quad v(x,y,t) = \widehat{v}(x,y,t), \quad p(x,y,t) = p_b(x) + \widehat{p}(x,y,t),$$
$$T(x,y,t) = T_b(y) + \widehat{T}(x,y,t), \quad \text{and} \quad \mu(x,y,t) = \mu_b(y) + \widehat{\mu}(x,y,t). \tag{12}$$

Substituting Eqs. (12) into Eqs. (5), (6) and (7), subtracting the base flow equations, and linearizing one obtains

$$\frac{\partial \widehat{u}}{\partial x} + \frac{\partial \widehat{v}}{\partial y} = 0\,, \tag{13}$$

$$\frac{\partial \widehat{u}}{\partial t} + u_b\frac{\partial \widehat{u}}{\partial x} + \widehat{v}\frac{du_b}{dy} = -\frac{\partial \widehat{p}}{\partial x} + \frac{\mu_b}{Re}\left(\frac{\partial^2 \widehat{u}}{\partial x^2} + \frac{\partial^2 \widehat{u}}{\partial y^2}\right) + \frac{1}{Re}\frac{d\mu_b}{dy}\left(\frac{\partial \widehat{u}}{\partial y} + \frac{\partial \widehat{v}}{\partial x}\right) + \frac{1}{Re}\frac{du_b}{dy}\frac{\partial \widehat{\mu}}{\partial y} + \frac{\widehat{\mu}}{Re}\frac{d^2 u_b}{dy^2}\,, \tag{14a}$$

$$\frac{\partial \widehat{v}}{\partial t} + u_b\frac{\partial \widehat{v}}{\partial x} = -\frac{\partial \widehat{p}}{\partial y} + \frac{2}{Re}\frac{d\mu_b}{dy}\frac{\partial \widehat{v}}{\partial y} + \frac{\mu_b}{Re}\left(\frac{\partial^2 \widehat{v}}{\partial y^2} + \frac{\partial^2 \widehat{v}}{\partial x^2}\right) + \frac{1}{Re}\frac{\partial \widehat{\mu}}{\partial x}\frac{du_b}{dy} \tag{14b}$$

$$\frac{\partial \widehat{T}}{\partial t} + u_b \frac{\partial \widehat{T}}{\partial x} + \widehat{v} \frac{dT_b}{dy} = \frac{1}{RePr} \left(\frac{\partial^2 \widehat{T}}{\partial x^2} + \frac{\partial^2 \widehat{T}}{\partial y^2} \right) . \tag{15}$$

The continuity equation can be eliminated by introducing a perturbation streamfunction $\widehat{\psi}$. It is further assumed that all disturbances have time and spatial dependence of the form (Drazin and Reid [8], Simpkins and Liakopoulos [9]),

$$(\widehat{\psi}, \widehat{p}, \widehat{T}, \widehat{\mu}) = [\phi(y), f(y), \theta(y), \Lambda(y)]\, e^{i\alpha(x-ct)} , \tag{16}$$

where α is the wavenumber, c is the complex disturbance velocity and ϕ, f, θ and Λ denote the disturbance amplitudes.

Substituting Eq. (16) into Eqs. (14) and (15), and eliminating the pressure perturbation terms by cross-differentiation, the stability governing equations become,

$$i\alpha Re\{(u_b - c)(\phi'' - \alpha^2\phi) - u_b''\phi\} = \mu_b(\phi^{IV} - 2\alpha^2\phi'' + \alpha^4\phi) + 2\mu_b'(\phi''' - \alpha^2\phi')$$

$$+ \mu_b''(\phi'' + \alpha^2\phi) + u_b'(\Lambda'' + \alpha^2\Lambda) + 2u_b''\Lambda' + u_b'''\Lambda \tag{17}$$

$$i\alpha RePr\{(u_b - c)\theta - \phi T_b'\} = (\theta'' - \alpha^2\theta) \tag{18}$$

where primes denote differentiation with respect to y.

Introducing disturbances in Eq. (8), expanding into Taylor series, and neglecting nonlinear terms (a procedure similar to the treatment of viscous, inelastic fluids in isothermal channel flows discussed by Pinarbasi and Liakopoulos [10]), one obtains a relationship between the amplitudes of viscosity and temperature fluctuations

$$\Lambda(y) = \beta(y)\theta(y) \qquad \text{where} \qquad \beta(y) = -\frac{C}{T_b^2}\, e^{\frac{1}{T_b}} . \tag{19}$$

Substituting Eq. (19) into Eq. (17), we derive two ODEs for $\phi(y)$ and $\theta(y)$, governing the stability of the flow. Note that Eq. (17) reduces to the classical Orr-Sommerfeld equation when $\mu_b = 1$ and $\theta = 0$, and it becomes identical to the stability equation derived by Potter and Graber [3] and Wazzan *et al.* [4] when $\theta = 0$.

The associated boundary conditions are,

$$\phi = \phi' = \theta = 0 \quad \text{at } y = 0 \text{ and } y = 1. \tag{20}$$

In this paper, we approach the stability question as a temporal stability problem, i.e. for an arbitrary positive real value of α, we obtain the complex eigenvalue c and the corresponding eigenfunctions ϕ and θ. The sign of c_i determines the stability of the flow, i.e., if $c_i > 0$ the flow is temporally unstable.

Method of Solution

The continuous eigenvalue problem formulated in the previous section is discretized using a Chebyshev collocation technique. The physical space $0 \le y \le 1$ is transformed to $-1 \le Y \le 1$ and the functions ϕ and θ are expanded in series of Chebyshev polynomials. Keeping $N+1$ terms in each series, $2(N+1)$ expansion coefficients are introduced as unknowns. The method of point collocation is applied to Eqs. (17) and (18) at

$$Y_j = \cos\left(\frac{\pi j}{N-3}\right), \quad j = 0, 1, 2 \ldots N-3.$$

The resulting $2(N-2)$ equations are supplemented by the discrete versions of the six boundary conditions, Eq. (20), to obtain $2(N+1)$ equations. This procedure results in a generalized matrix eigenvalue problem of the form $Ax = cBx$ which is solved using the IMSL routine *dgvlcg*. All calculations were performed in double-precision arithmetic on an IBM RS/6000 workstation. Up to $N = 80$ terms were used to achieve convergence in the computation of eigenvalues. $N = 50$ gives satisfactory convergence for $Re < 10{,}000$ and $\alpha < 1.5$, but larger N is required at higher values of Re and α.

The computer implementation of the procedure outlined above was tested by comparison with Orszag's [2] stability results for isothermal Poiseuille flow. Since the choice of coordinate system, characteristic length and definition of Reynolds number in Orszag's paper differ from those adopted in the present study, attention must be given in establishing the correspondence. In terms of Orszag's definitions, the eigenvalues for ($Re = 10{,}000$, $\alpha = 1$) and ($Re = 5772.22$, $\alpha = 1.02056$) are

$$c = 0.23752649 + i\,0.00373967 \text{ and } c = 0.26400174 + i\,5.9\text{x}10^{-10}$$

respectively. For these parameters, our code gives for $\Delta\bar{T} \to 0$

$$c = 0.23752647 + i\,0.00373967 \text{ and } c = 0.26400176 + i\,3.7\text{x}10^{-9}.$$

Results and Discussion

The primary goal of this study is to investigate the influence of temperature dependent viscosity on the stability of plane Poiseuille flow. The results presented in this section are obtained for water. The dependence of viscosity of water on temperature is adequately modeled by Eq. (4). The constants in Eq. (4) are $\bar{d} = 1796.1$ K and $C = 0.00224266$. Fig. 1 shows the dimensionless velocity and viscosity profiles for $Re = 10{,}000$ and $\Delta\bar{T} = \bar{T}_h - \bar{T}_c = 0$ K, 11.1 K, 55.6 K and 111.1 K. Since the viscosity of water decreases with increasing temperature, the dimensionless velocity profiles become skewed as $\Delta\bar{T}$ increases and the location of maximum speed is shifted towards the upper hot wall.

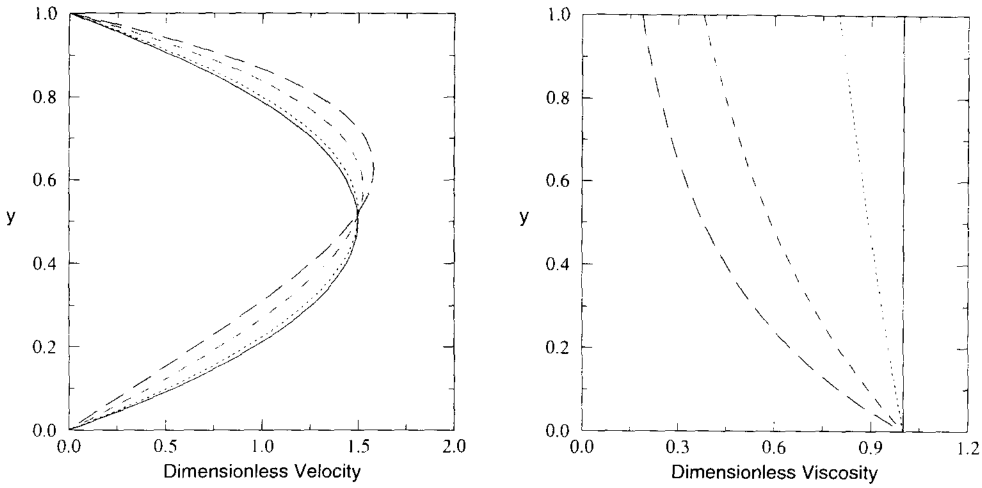

FIG. 1
Velocity and Viscosity Profiles for Various Wall Temperature Differences. Re=10,000.
____ $\Delta\bar{T}$=0 K .... $\Delta\bar{T}$=11.1 K --------- $\Delta\bar{T}$=55.6 K _ _ _ $\Delta\bar{T}$=111.1 K

The effect of temperature dependent viscosity on stability can be seen in Fig. 2. In this figure, the imaginary part of the eigenvalue c is given as function of the wavenumber α for two Reynolds numbers. For the variable viscosity case, $Pr = 7.0$ and $\Delta\bar{T} = 111.1$ K. The flow becomes unstable whenever c_i exceeds zero. For variable properties, the flow becomes unstable at $Re_c = 3035.805$, $\alpha_c = 2.154$. For this value of Reynolds number, however, the flow is still stable if viscosity variations are neglected, as the broken-line curve (classical Orr-Sommerfeld problem) indicates. For the constant property case, flow instability is not observed until Reynolds number is increased to 7696.29 (see Fig. 2b). The value of α_c for this Reynolds number is 2.041. It is seen that the inclusion of viscosity variations with temperature is extremely important for the determination of flow instabilities in plane nonisothermal Poiseuille flow; without their inclusion, the critical Reynolds number is overestimated.

Table 1. shows the effect of wall temperature difference $\Delta\bar{T}$ on the critical Reynolds and wavenumbers with and without viscosity and temperature fluctuations. The critical Reynolds number, Re_c, is defined as the smallest value of Reynolds number for which an unstable eigenmode exists. All results listed in Table 1 are obtained for $N = 70$. For constant property ($\Delta\bar{T} = 0$), $Re_c = 7696.29$ and $\alpha_c = 2.041$ which agrees well with the results of Orszag [2]. As $\Delta\bar{T}$ increases, Re_c decreases rapidly while α_c increases slightly. When viscosity and temperature fluctuations are included, Re_c increases slightly and α_c decreases slightly compared to the values calculated when viscosity and temperature fluctuations are neglected.

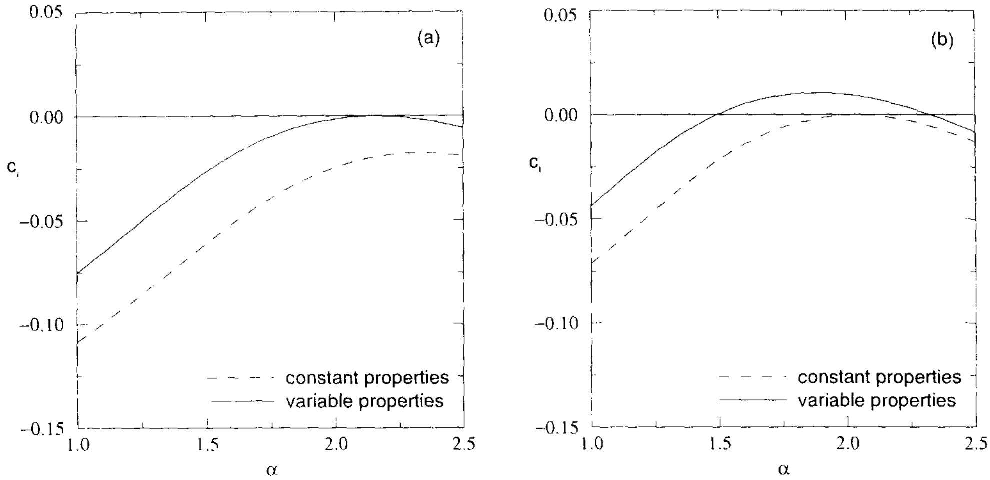

FIG. 2
Imaginary Part of Complex Disturbance Velocity for Constant and Variable Viscosity.
Pr=7.0, $\Delta\bar{T}$=111.1 K. (a) Re=3035.805 (b) Re=7696.29 .

TABLE 1
Effect of Wall Temperature Difference $\Delta\bar{T}$ on the Critical Reynolds and Wave Numbers.
$Pr = 7.0$, $N = 70$.

	Re_c	α_c	c
	constant properties		
$\Delta\bar{T} = 0\,K$	7696.29	2.041	0.39599215 + i 7.9318348x10^{-8}
	variable viscosity, viscosity & temperature fluctuations neglected		
$\Delta\bar{T} = 11.1\,K$	6807.21	2.043	0.40075724 + i 3.0362995x10^{-8}
$\Delta\bar{T} = 55.6\,K$	4619.2	2.076	0.40061146 + i 3.3968352x10^{-8}
$\Delta\bar{T} = 111.1\,K$	2977.3	2.161	0.40743483 + i 5.3442784x10^{-8}
	variable viscosity, viscosity & temperature fluctuations included		
$\Delta\bar{T} = 11.1\,K$	6893.0	2.043	0.39597355 + i 1.0695710x10^{-8}
$\Delta\bar{T} = 55.6\,K$	4721.1	2.072	0.39544820 + i 4.2346046x10^{-9}
$\Delta\bar{T} = 111.1\,K$	3035.805	2.154	0.40253716 + i 1.1042853x10^{-7}

The influence of wall temperature difference $\Delta\bar{T}$ on the shear flow instability is shown in Fig. 3 in the (α, Re) plane. The map is formed for three different values of $\Delta\bar{T}$: 11.1 K, 55.6 K and 111.1 K neglecting viscosity and temperature fluctuations. As $\Delta\bar{T}$ increases, the marginal stability curves shift to the left, thus indicating a more unstable flow situation. This agrees well with the results of Potter and Graber [3] who neglected temperature and viscosity fluctuations in their study. In order to assess the effect of temperature and viscosity fluctuations on stability, we recalculated the stability curve for $\Delta\overline{T} = 111.1$ K retaining viscosity and temperature fluctuations in the formulation. Comparison is made in Fig. 4 for Prandtl number $Pr = 7.0$. The two curves, with and without fluctuations, are almost indistinguishable. In Fig. 5, the imaginary part of the eigenvalue c is plotted versus the wavenumber α for $Re = 500,\ 1,000,\ 5,000$ and $\Delta\overline{T} = 111.1$ K. The broken-line curves correspond to the full formulation that includes viscosity and temperature fluctuations. The results indicate that temperature and viscosity fluctuations influence the complex disturbance velocity for values of wavenumber $\alpha \leq 1.5$, but their effect on the instability of the flow is very small. Furthermore, Fig. 5 indicates that the effect of temperature and viscosity fluctuations diminishes as Re increases. This observation is in agreement with the two-dimensional boundary layer stability results of Herwig and Schäfer [5].

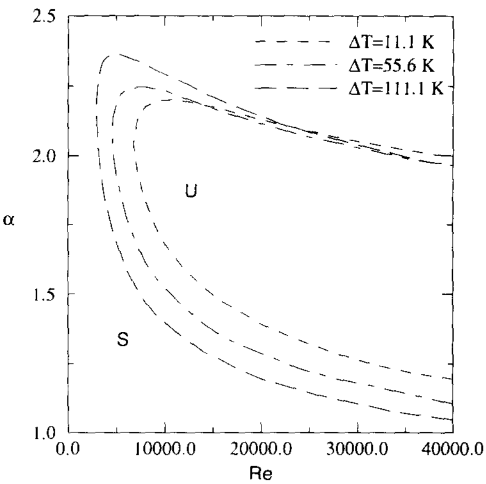

FIG. 3
Effect of Wall Temperature Difference $\Delta\overline{T}$ on Stability. S: stable, U: unstable.

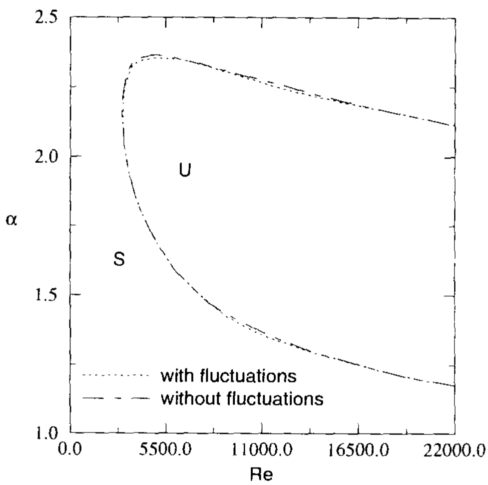

FIG. 4
Marginal Stability Curves for Pr=7.0 and $\Delta\overline{T}$=111.1 K

Finally, in Fig. 6 we present the influence of Prandtl number on flow instability. The relation $c_i(\alpha)$ is given for Re = 500, 1,000, 5,000 and $\Delta\overline{T} = 111.1$ K. Both temperature and viscosity fluctuations are included in the calculations. The Prandtl number takes the values of 3.56 and 7.0, corresponding to reference wall temperature $\overline{T}_r = 50$°C and 20°C, respectively. It is seen that although Pr affects the imaginary part of the complex disturbance velocity for small values of Re and α, it has a small effect on flow instability.

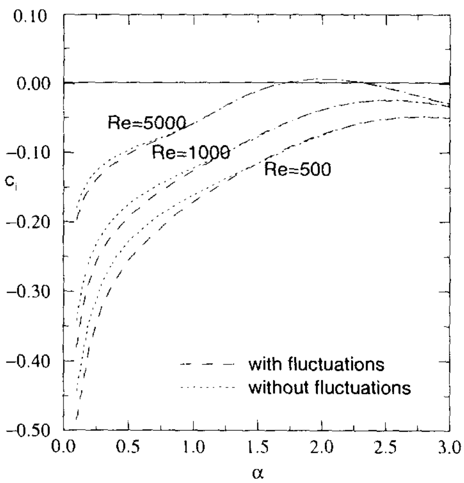

FIG. 5
Imaginary Part of Complex Disturbance Velocity for Various Reynolds Numbers. Pr=7.0, $\Delta\bar{T}$=111.1 K.

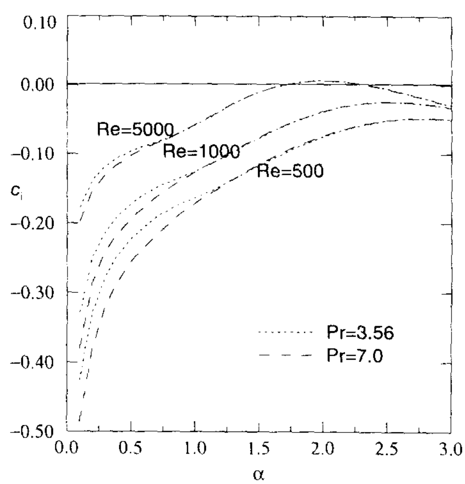

FIG. 6
Imaginary Part of Complex Disturbance Velocity for Various Prandtl and Reynolds Numbers. $\Delta\bar{T}$=111.1 K.

Conclusions

The influence of temperature dependent viscosity on the stability of a Newtonian liquid flowing between two parallel plates is investigated. Assuming a linear temperature profile and an exponential viscosity law, a two-dimensional, linear stability analysis is performed. The primary flow is affected through the variation of viscosity with temperature. In addition to velocity and pressure fluctuations, temperature and viscosity fluctuations are also considered and the coupled momentum-energy stability equations are solved simultaneously. It is found that the critical Reynolds number decreases considerably compared to isothermal Poiseuille flow. An imposed wall temperature difference, $\Delta\bar{T}$, is always destabilizing and the destabilizing effect is more pronounced as $\Delta\bar{T}$ increases. The influence of Prandtl number, temperature fluctuations and viscosity fluctuations on the flow stability/instability is found to be small. However, their influence on the margin of stability , as measured by the imaginary part of the complex velocity of the disturbance, is appreciable for small wavenumbers ($\alpha \leq 1.2$).

Acknowledgements

This work was partially supported by NASA/LeRC under contract No. NAG3-1632.

Nomenclature

c	complex disturbance velocity
c_p	specific heat at constant pressure
C, d	viscosity law constants
k	thermal conductivity
l	channel width
p	pressure
Pr	Prandtl number
Re	Reynolds number
t	time
T	temperature
u, v	x-, y-velocity components
x, y	Cartesian coordinates
Greek Letters	
α	wavenumber
μ	fluid viscosity
ρ	fluid density
τ	deviatoric stress tensor
$\Delta\bar{T}$	wall temperature difference
ψ	streamfunction
Subscripts/Superscripts	
b	base flow
m	mean value
o	reference value
c	critical value, cold wall
h	hot wall

References

1. L. H. Thomas, Phys. Review, 91, 780 (1953).
2. S. A. Orszag, J. Fluid Mech., 50, 689 (1971).
3. M. C. Potter, and E. J. Graber, NASA TN D-6027 (1970).
4. A. R. Wazzan, G. Keltner, T. T. Okamura, and A. M. O. Smith, Phys. Fluids, 15, 2114 (1972).
5. H. Herwig, and P. Schäfer, J. Fluid Mech., 243, 1 (1992).
6. P. Schäfer, and H. Herwig, Int. J. Heat Mass Transfer, 36, 2441 (1993).
7. M. Abramowitz, and I. A. Stegun, Handbook of Mathematical functions, Dover Publ., New York (1965).
8. P. G. Drazin, and W. H. Reid, Hydrodynamic Stability, Cambridge Univ. Press, Cambridge, Mass. (1981).
9. P. G. Simpkins, and A. Liakopoulos, Stability of Convective Flows, ASME Press, New York (1992).
10. A. Pinarbasi, and A. Liakopoulos, J. Non-Newtonian Fluid Mech., 57, 227 (1995).

Received April 21, 1995